Enrollment No: ____________________ Exam Seat No: ____________________

C.U.SHAH UNIVERSITY

Summer Examination-2020

Subject Name: Advanced Calculus

Subject Code: 4SC03ADC1 **Branch: B.Sc. (Physics)**

Semester: 3 **Date: 27/02/2020** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: **(14)**

a) If $u = \cot^{-1}(x+y)$ then $u_x - u_y =$ ______. (02)

b) If $z = f(u,v)$, $u = g(x,y)$ and $v = h(x,y)$ then $\frac{\partial z}{\partial x} =$ ________. (02)

c) If $f(x,y,z) = \frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}}$ then find the value of $xf_x + yf_y$. (02)

d) Define: Jacobian (01)

e) Write the relation between beta and gamma function. (01)

f) $\Gamma(n)\Gamma(1-n) =$ ______. (01)

g) Find: $\Gamma(3.5)$ (02)

h) Define: Point of inflection (01)

i) Define: Concave upwards, Concave downwards (02)

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions **(14)**

a) If $u = \tan^{-1}\left(\frac{y}{x}\right)$ then show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. (05)

b) Determine whether $\lim_{(x,y)\to(0,0)} \frac{x^2 + y}{x^4 + y^2}$ exit or not? If they exist find the value of limit. (05)

c) If $y = f(x+at) + g(x-at)$ then prove that $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$. (04)

Q-3 Attempt all questions **(14)**

a) Find the extreme value of $f(x,y)=x^2y-xy^2-4x^2-4y^2+4xy$. (05)

b) Evaluate: $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}dx \cdot \int_0^1 \frac{dx}{\sqrt{1+x^4}}$ (05)

c) If the function $z=f(x,y)$ possesses the first order derivatives in the domain D and $x=g(t), y=h(t)$ also possesses its first order partial derivative then show that $\frac{dz}{dt}=\frac{\partial z}{\partial x}\frac{dx}{dt}+\frac{\partial z}{\partial y}\frac{dy}{dt}$. (04)

Q-4 Attempt all questions **(14)**

a) If u is a homogeneous function of degree n in the variable x and y then $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u$. (05)

b) If $x=r\cos\theta, y=r\sin\theta$ then find the value of $\left[x\left(\frac{\partial x}{\partial r}\right)_\theta+y\left(\frac{\partial y}{\partial r}\right)_\theta\right]^2$. (05)

c) If $z=f\left(e^u+e^{-v}, e^{-u}+e^v\right)$ then prove that $\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}=x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}$. (04)

Q-5 Attempt all questions **(14)**

a) State and prove Taylor's series for function of two variables. (07)

b) Evaluate: $\int_0^\infty x^n \cdot e^{-a^2x^2}\, dx$ (04)

c) Find the maximum value of $x^2y^3z^4$ given that $2x+3y+4z=0$ by using Lagrange's method. (03)

Q-6 Attempt all questions **(14)**

a) If $u=\tan^{-1}\left(x^2+2y^2\right)$ then prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\sin u\cdot\cos 3u$. (05)

b) If $u=x+y+z,\ v=x^2+y^2+z^2,\ w=x^3+y^3+z^3$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. (05)

c) Prove that $B(m+2,n-2)=\frac{m(m+1)}{(n-1)(n-2)}B(m,n)$. (04)

Q-7 Attempt all questions **(14)**

a) If $u=f(r)$, where $r^2=x^2+y^2$ then prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$. (05)

b) Find the asymptote of the following: (05)

i) $4x^2+9y^2=16x^2y^2$ ii) $y^2\left(a^2-x^2\right)=x$

c) Prove that $\Gamma(n)\Gamma\left(n+\frac{1}{2}\right)=\frac{\sqrt{\pi}\cdot\Gamma(2n)}{2^{2n-1}}$. (04)

Q-8 Attempt all questions **(14)**

a) Evaluate: $\int_0^1 x^5\left(1-x^3\right)^3 dx$ (05)

b) Find the point of inflection of the following: (05)

i) $f(x)=(x-2)^3(x-3)^2$

ii) $f(x)=x^5-5x^4+5x^3-1$

c) Expand $\frac{1}{xy}$ in powers of $(x-1)$ and $(y+2)$ up to second order terms. (04)